Lesson Title: Estimation – Make Reasonable Estimates (Multiply and Divide Decimals)

Concept/s in Focus:

- Estimates can make complex computations easier to manage.
- Estimates are not intended to be 'correct' to reduce a focus on exactness.
- Estimation language includes terms like about, close to, a little less/more than and between.
- Rounding numbers to a particular place value is a valuable skill when estimating.
- Rounding decimals can make numbers into whole numbers or decimals.
- Estimation strategies provide ways to approach estimation situations.
- Some estimation strategies work better to solve problems involving particular operations.
- Multiplying by numbers less than 1 will result in a number less than the same multiplication involving whole numbers.
- Dividing by numbers less than 1 will result in a number greater than the same division involving whole numbers.
- Multiplication is commutative. Swapping the order of the factors in a particular multiplicative situation will result in the same total e.g. 0.5x3 = 1.5 = 3x0.5 (half of 3 = 3 halves)

Introduction / Teacher Background Information:

Estimating answers to multiplication and division computations can be done using the same strategies that work for estimating addition and subtraction computations e.g. Front End and Rounding. For division, the Compatible numbers strategy is the major focus. Dividing 287 by 6 will be easier if students can recognise that 6x50=300 which is close to 287 so 50 would be a reasonable estimate. When the operation involves decimals, the basic facts still apply e.g. 28.7 ÷6 will be close to 5. For this strategy students need a sound understanding of their multiplication and division basic facts and their extensions into larger numbers. Estimation activities can help students to see a need for knowing these facts. Estimating requires a sound understanding of the operations involved. Computations involving decimals require a sound understanding of decimal place value. The focus of this lesson is NOT for students to work out exact answers. The computations used in this lesson are quite challenging – multiplying by decimals and dividing decimals by whole numbers. The more challenging computations should encourage students to simplify them by using their multiplication and division fact knowledge and estimate rather than completing the computations.

Australian Curriculum links: AC9M6N06

Resources:

Whole Class Activity:

- PowerPoint presentation – Estimate Multiplication and Division (Decimals)
- 1 copy of the Estimation – Multiply and Divide Decimals Scoresheet.
- A small whiteboard per student or notebook.

Hands-On Activity:

- 1 copy of the Calculator Snooker Game Board per pair of students.
- 1 copy of the Calculator Snooker Instructions either per pair of students or one copy to project.
- An online video of snooker being played (optional)
- Spare copies of the Calculator Snooker Board for students to play the division version.
- A calculator per pair of students

Independent Activity:

- 1 copy of the Estimation (Multiply and Divide Decimals) Worksheet per student.

Acknowledgement to C.Lovitt and D.Clarke (1992) Maths Curriculum Teaching Program (MCTP) Curriculum Corporation (p. 433-435) and M. Swan 1984 (Shell Centre for Mathematical Education)

Whole Class Activity:

- Preparation: Run through the Estimate Multiplication and Division (Decimals) PowerPoint Presentation in Slideshow mode to test the animations and become familiar with the presentation structure.

- Have the students sit at desks or in a position where they can see the PowerPoint presentation.
- Ensure the students have a small whiteboard, book or piece of paper to write their estimated answers so they can be kept for review later in the lesson.
- The PowerPoint has an introduction slide asking a couple of questions about estimating.
- Project these questions one by one and invite a number of students to share their thoughts i.e.

What does it mean to estimate?

- Invite a number of students to explain what they think estimating is. Focus on answers that include the idea of it being an educated guess, not just a random guess.
- Project the example computation 26 x 5. The slide has animations.

Will the answer be more or less than 100?

- Have the students write whether their response to the question on their whiteboard / page.

How did you estimate whether the answer would be more than 100 or not?

- Invite a number of students to share how they worked out whether the answer to 26x5 would be more or less than 100.
- Listen for mention of using a Compatible Numbers Strategy (known number facts and extensions) e.g. knowing 25x4=100 or 20x5=100 or 30x5=150 so 26x5, so 26x5 must be larger than 100.
- Another strategy students may use is a Rounding Strategy where numbers are changed to be nearly manageable numbers (usually multiples of ten) e.g. considering 20x5 and/or 30x5 would be examples of using rounding.
- Continue to the next slide which shows the same computation as a division.

This time the computation to estimate is a division. What number will the answer be close to? We don't want the exact answer. We are looking for a reasonable estimate.

- Invite several students to explain their reasoning.
- Listen for use of Compatible Numbers or Rounding again e.g. 25÷5=5 or 20÷5=4 or 30÷5=6 and how these known facts helped the student decide that the answer would be about 5 or slightly more than 5 because 26 is more than 25 and / or less than 30.
- The focus is for students to choose numbers easy to work with to find the estimate quite quickly.

- The PowerPoint then has 5 multiplications involving decimals for the students to practise estimating (2.5x7, 16x2.8, 4x19.2, 20.5x15, 5.2x2.3). The intention is for them to see that using multiplication facts can help them to find estimates and can make computations easier, including if they involve decimals.
- Show the PowerPoint slide with all 5 multiplications. The slide is animated so they appear, then disappear on the next mouse click. Show each multiplication for about 5 seconds (or a reasonable time but not long enough for calculation of an exact answers).
- The PowerPoint presentation then has a slide to review each multiplication and provides a scoring rubric according to how close their estimate was to the actual answer as a number range to provide some impetus to try to be as close to the actual answer as they can.

- Note: Because the answers are decimals, the scores have overlapping ranges otherwise there would always be possible answers between the ranges given. Allow students whose estimate is on these overlaps to choose the higher score.

What strategies did you use to estimate the answers to these multiplications?

- Listen for mention of Rounding or use of Compatible Numbers estimation strategies.

- Continue the presentation. The next section provides 5 division computations involving decimals but related to division basic facts (25.2÷4, 43.8÷5, 77.1÷9, 98.6÷3, 24.2÷0.5). The last division is a challenge and provides a decimal divided by a familiar decimal.
- Show the divisions one by one (as before) and have the students estimate an answer to each.
- Show the scoring slide for each division and have the students work out their score for each.
- Note: Because the answers are decimals, the scores have overlapping ranges otherwise there would always be possible answers between the ranges given. Allow students whose estimate on these overlaps to choose the higher score.

What strategies did you use to estimate the answers to these divisions?

- Listen for mention of using Compatible Numbers strategies based on known multiplication / division basic facts.

Hands On Activity:

- Acknowledgement. Ideas for this section of the lesson are from the Calculator Snooker lesson described in Lovitt, C & Clarke, D (1992) *Mathematics Curriculum Teaching Program (Vol 2, pp 433-435)* Curriculum Corporation.

- The students will need to work with a partner for this section of the lesson as the activity is a game for 2 players. The game can be played with groups of 3 but the score sheet is designed for 2 so they will need an extra page.
- The activity in this section of the lesson is from the Mathematics Curriculum Teaching Program (MCTP) authored by Charles Lovitt and Doug Clarke (1992) which also references Malcolm Swan of the Shell Centre for Mathematics Education (1984). The division modification was created for this lesson.

Have you heard of the game Snooker? What do you know about it?

- Invite several students to describe what they know of the game. It is different to the possibly more familiar game of 'Pool' but is played on a 'pool table' with cues.
- If desired a video of the game of snooker could be viewed to provide context. A web search for 'snooker' or 'Pot Black' has clips of the game being played.
- Ensure the students understand the concept of 'potting' balls and that the game progresses by 'potting' a red ball followed by any coloured ball, then a red ball… until all red balls are gone. Then the coloured balls are 'potted' in order – Yellow, Green, Brown, Blue, Pink, Black.

You are going to play a version of snooker with your calculator. You will use your estimation skills to get answers on your calculator that are close to a particular number to 'pot' the different colour balls.

- The Instructions for the game can be printed for each pair of students or projected for all to see and discuss together.
- Provide each pair of students with a copy of the Calculator Snooker Game Board and ensure each pair of students has a calculator.

- To ‘pot’ the balls in Calculator Snooker, students need to perform a multiplication calculation (and in the modified version of the game a division calculation) so the first digit matches the list for the colour they are after. This will require estimation and the strategies discussed in the Whole Class section of the lesson can be used.
- Involving decimals will be a good strategy for this game. The answers will also end up with decimals (especially the division game).
- It could be helpful to model ‘potting’ one ball by having a student give a 2-digit start number.
- Write the start number on the whiteboard.

To start the game I need to ‘pot’ a red ball first. To ‘pot’ a red ball I need to do a multiplication calculation using this start number and get an answer that has a 1 as the first digit.

- Use a ‘think aloud’ strategy to talk through an estimation process for the given start number e.g. if the start number was 78 thinking could be ‘I know that 2x70 is 140 so I might try multiplying by 2 but I think that will be too high. I am going to try a number a bit smaller than 2.. 1.6.
- Write the computation chosen on the whiteboard i.e. 1.6 x 78.
- Complete the calculation on the calculator by entering ‘x1.6’ to the 78 already entered.
- Announce the answer and record it on the whiteboard i.e. 78 x 1.6 = 124.8. The red ball is ‘potted’.
- As the ‘potting’ was successful the players need to cross off one of the red balls (these are not replaced during the game).
- The same player continues as long as balls are ‘potted’. The next target is a coloured ball.
- Using the result of the previous calculation i.e. 124.8 the player uses multiplication to ‘pot’ any colour ball they choose. Black is worth the most points and needs an answer with a 7 as the first digit. Potting any ball next is OK though e.g. try for 7 by multiplying 124.8 by 6.. 748.8. Then the player has to try for a red ball again (first digit 1)..
- Model the game until the class understands.
- It is important the players don’t clear the calculator. To avoid hassles if this happens, have each player record their calculation on the Game Board sheet.
- Ensure they students understand the coloured balls are ‘replaced’ or always available until all the red balls have been ‘potted’.
- Set the students to play the game as soon as the rules are understood (to learn by doing 😊)
- Move around the classroom as the students play listening to their estimation strategies to aim to get particular digits in the front of their answers.
- Identify any interesting strategies or specific calculations involving decimals that will be good to discuss as a whole class when the activity is complete.
- Ask questions to guide students who are unsure what multiplier to choose e.g.

What colour ball are you trying to ‘pot’? What number is on your calculator now that will be your start number? What are you thinking you might try? Why do you think that might be a good number? What is the first digit of the number in your calculator now? What could you multiply a number with that digit at the start by to get a start digit you want? Think about the multiplication facts using the first digit.

- When the students have played the game for a reasonable amount of time or they have finished a game by ‘potting’ all the balls, gather the class and discuss strategies they used.
- Invite pairs of students who had interesting computations and estimation strategies to share their reasoning and the results they achieved.

Did your estimation improve as you kept playing the game?

- Invite several students to comment on their estimation strategies and how they developed.
- If desired, the division version of the game can be explained and students can play this game.
- The multiplication and division versions of this game can be used in rotations or as fast finisher activities at other times.

Independent Activity:

- Provide each student with a copy of the Estimation – (Multiply and Divide Decimals) Worksheet.
- This worksheet provides 5 multiplications and 5 division computations involving decimals.
- The students are asked to provide a range that the answer will be between and to then show their reasoning and an estimate of the answer for each computation.

This worksheet has multiplications and divisions for you to ESTIMATE the answers to.
Your first task is to think of a range of numbers you think the answer will be between.
Then you decide on your estimate. Show all your thinking in the space for each computation.

- Set the students to work on the estimations.
- Move around the classroom as the students work observing their processes and asking questions to identify understandings e.g.

How did you work out the estimation for this multiplication? What did you think about?
How are you going to show your thinking to work out the actual answer to this multiplication?

- When the students have completed the worksheet, it can be collected (if being used for assessment purposes) or the responses to each multiplication can be shared and strategies for estimation and calculation can be discussed as a class.

Understandings to look for:

- Students who can round decimals to nearby whole numbers and decimals.
- Students who can change numbers in computations to create compatible numbers that allow an estimate to be made easily.
- Students who can estimate answers to multiplication and division computations involving decimals.
- Students who understand that multiplying by a decimal less than one will result in a smaller answer than if they had multiplied by the same number as a whole number.